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Møller's Four-Momentum of Electric and Magnetic Black Holes

Figen Binbay · Nurettin Pirinccioglu · Mustafa Saltı · Oktay Aydogdu

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Abstract In order to evaluate energy and momentum components associated with two different black hole models, i.e. the electric and magnetic black holes, we use the Møller energy-momentum prescriptions both in Einstein's theory of general relativity and the teleparallel gravity. We obtain the same energy and momentum distributions in both of these different gravitation theories. The energy distribution of the electric black hole depends on the mass M and the magnetic black hole energy distribution depends on the mass M and charge Q. In the process, we notice that (a) the energy obtained in teleparallel gravity is also independent of the teleparallel dimensionless coupling parameter, which means that it is valid not only in teleparallel equivalent of general relativity but also in any teleparallel model, (b) our results also sustains the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given spacetime, and (c) the results obtained support the viewpoint of Lessner that the Møller energy-momentum complex is a powerful concept of energy and momentum.

Keywords Møller energy · Electric and magnetic black holes · Teleparallel gravity

F. Binbay (✉) · N. Pirinccioglu
Department of Physics, Faculty of Art and Science, Dicle University, 21280, Diyarbakir, Turkey
e-mail: figenb@dicle.edu.tr

N. Pirinccioglu
e-mail: npirinc@dicle.edu.tr

M. Saltı · O. Aydogdu
Physics Department, Art and Science Faculty, Middle East Technical University, 06531, Ankara, Turkey

M. Saltı
e-mail: musts6@yahoo.com

O. Aydogdu
e-mail: oktay231@yahoo.com

1 Introduction

The formulation of energy-momentum distribution was initiated by Einstein [1]. After that a large number of prescriptions of the gravitational energy, momentum and angular momentum have been proposed. Some of them are coordinate-independent and others are coordinate-dependent. There lies a dispute on the importance of non-tensorial energy-momentum complexes whose physical interpretations have been questioned by a number of physicists, including Weyl, Pauli and Eddington. Also, there exists an opinion that the energy-momentum pseudotensors are not useful to find meaningful results in a given geometry. Chang, Nester and Chen [2] obtained that there exists a direct relationship between quasilocal and pseudotensor expressions, since every energy-momentum pseudotensor is associated with a legitimate Hamiltonian boundary term. Ever since the Einstein's energy-momentum complex, used for calculating energy and momentum in a general relativistic system, many attempts have been made to evaluate the energy distribution for a given spacetime [3–12]. Except for the Møller's definition these formulations only give meaningful results if the calculations are performed in *quasi-Cartesian* coordinates. Møller proposed a new expression for energy-momentum complex which could be utilized to any coordinate system. Next, Lessner [13] argued that the Møller prescription is a powerful concept of energy-momentum in general relativity.

Virbhadra [14], using the energy and momentum complexes of Einstein, Landau–Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr–Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in the Penrose energy-momentum formulation. In literature, there are several papers on the calculation of the energy-momentum distribution of the universe by using energy-momentum complexes [15–43].

Recently, the problem of energy-momentum localization has also been considered in teleparallel gravity [44, 45]. Møller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment of the energy-momentum complex than does general relativity. Therefore, we have also applied the super-potential method by Mikhail et al. [46] to calculate the energy of the central gravitating body. In Gen. Rel. Grav. 36, 1255 (2004); Vargas, using the definitions of Einstein and Landau–Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann–Robertson–Walker spacetimes. There are also new papers on the energy-momentum problem in teleparallel gravity. The authors obtained the same energy-momentum for different formulations in teleparallel gravity [47–58].

The paper is organized as follows. In Sect. 2, we give two different black hole solutions which will be considered to obtain energy and momentum associated with them. Next, in Sect. 3, we introduce the energy-momentum definitions of Møller, and then using it calculate the energy-momentum distributions in electric and magnetic black hole solutions both in general relativity and teleparallel gravity. Finally, Sect. 4 is devoted to final comments.

Notations and conventions: $c = h = 1$, metric signature $(-, +, +, +)$, Greek indices run from 0 to 3 and, Latin ones from 1 to 3. Throughout this paper, Latin indices $(i, j, k, \ldots)$ number the vectors, and Greek indices $(\mu, \nu, \alpha, \ldots)$ represent the vector components.

2 Stringy Black Holes

The low-energy effective theory largely resembles general relativity with some *matter* fields as the dilaton, axion etc. [59, 60]. A main property of the low-energy theory is that there are two different frames in which the features of the spacetime may look very different. These two frames are the Einstein frame and the string frame, and they are related to each other by a conformal transformation ($g^E_{\mu\nu} = e^{-2\Phi} g^S_{\mu\nu}$) which involves the massless dilaton field as the conformal factor. The string *sees* the string metric. Many of the important symmetries of the string theory also rely on the string frame or the Einstein frame [61].

The action integral for the Einstein–dilaton–Maxwell (EDM) theory is given as

$$S^{\text{EDM}} = \int d^4x\sqrt{-g}e^{-2\phi}\left[R + 4g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\xi}g^{\nu\zeta}F_{\mu\nu}F_{\xi\zeta}\right]. \tag{1}$$

Varying with respect to the line-element, dilaton and Maxwell fields we get the field equations for the theory given as

$$\frac{1}{2}R_{\mu\nu} = F_{\mu\xi}F^\xi_\nu - \nabla_\mu\nabla_\nu\phi, \tag{2}$$

$$\nabla^\nu(e^{-2\phi}F_{\mu\nu}) = 0, \tag{3}$$

$$\nabla^2\phi - (\nabla\phi)^2 + \frac{R - F^2}{4} = 0. \tag{4}$$

These equations are also the β function equations for a worldsheet sigma model obtained by imposing quantum conformal invariance and setting β functions to zero. Without the Maxwell field we have essentially a Brans–Dicke type theory [10] with Brans–Dicke parameter explicitly set $\omega = -1$.

2.1 The Electric Black Hole

In the string frame that is actually similar to the Brans–Dicke frame in the well-known Jordan–Brans–Dicke theory, the metric and matter fields which solve the dilaton–Maxwell–Einstein field equations (2–4) to yield the electric black hole are given as [59]

$$ds^2 = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{2M\sinh^2\alpha}{r}\right)^{-2}dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{5}$$

and

$$A_t = -\frac{M\sinh\alpha}{\sqrt{2}(r + 2M\sinh^2\alpha)}, \tag{6}$$

$$e^{-2\phi} = 1 + \frac{2M\sinh^2\alpha}{r}. \tag{7}$$

Here, α contains charge parameter Q [59, 62], and M is the mass of the electric black hole. The geometry has a horizon at $r = 2M$ and a singularity at $r = 0$.

For the metric describing the electric black hole, the non-vanishing components of the Einstein tensor $G_{\mu\nu}$ ($\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid of density ρ and pressure p) are

$$G_{11}=4\sinh^2\alpha\frac{1}{r^2[-1+\frac{r}{M}+\cosh 2\alpha]}, \tag{8}$$

$$G_{22}=2\sinh^2\alpha\frac{[-M+4r-\frac{r^2}{M}+(M+r)\cosh 2\alpha]}{r[\frac{r}{M}+2\sinh\alpha]^2}, \tag{9}$$

$$G_{33}=2\sinh^2\alpha\sin^2\theta\frac{[-M+4r-\frac{r^2}{M}+(M+r)\cosh 2\alpha]}{r[\frac{r}{M}+2\sinh\alpha]^2}. \tag{10}$$

Next, for the spacetime of electric black hole (EBH), $g_{\mu\nu}$ is defined by

$$\begin{pmatrix} \left(1-\frac{2M}{r}\right)\left(1+\frac{2M\sinh^2\alpha}{r}\right)^{-2} & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2M}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}, \tag{11}$$

and its inverse $g^{\mu\nu}$ is

$$\begin{pmatrix} \left(1-\frac{2M}{r}\right)^{-1}\left(1+\frac{2M\sinh^2\alpha}{r}\right)^{2} & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2M}{r}\right) & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin^2\theta} \end{pmatrix}. \tag{12}$$

2.2 The Magnetic Black Hole

In the string frame, the dual solution known as the magnetic black hole is obtained by multiplying the electric metric in the Einstein frame by a factor $e^{-2\phi}$ (*here note the sign of* ϕ). In a more generalized sense this is the S-duality transformation which changes $\phi\to-\phi$ and thereby inverts the strength of the string coupling. Also, recall that the magnetic and electric solutions are the same if one looks from the Einstein frame.

Therefore, the magnetic black hole metric is given by [59]

$$ds^2=\frac{(1-\frac{2M}{r})}{(1-\frac{Q^2}{Mr})}dt^2-\left(1-\frac{2M}{r}\right)^{-1}\left(1-\frac{Q^2}{Mr}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2).$$

Here, M and Q are mass and charge of the magnetic black hole, respectively. For this black hole model, the non-vanishing components of the Einstein tensor $G_{\mu\nu}$ are

$$G_{11}=\frac{Q^2}{r}\frac{1}{(Mr-Q^2)(2M-r)}, \tag{13}$$

$$G_{00}=\frac{Q^2}{r^3}\frac{(r-2M)}{(Mr-Q^2)}. \tag{14}$$

And the metric tensor $g_{\mu\nu}$ of the magnetic black hole (MBH) is defined by

$$\begin{pmatrix} \frac{1-\frac{2M}{r}}{1-\frac{Q^2}{Mr}} & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2M}{r}\right)^{-1}\left(1-\frac{Q^2}{Mr}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}, \tag{15}$$

and its inverse $g^{\mu\nu}$ is

$$\begin{pmatrix} \frac{1-\frac{Q^2}{Mr}}{1-\frac{2M}{r}} & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right) & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin^2\theta} \end{pmatrix}. \tag{16}$$

We think that the other important and challenging problems related to these black hole solutions are those of their energy distributions. We consider that a good manner to evaluate the energy associated with a black hole solution in string theory is the one which uses the Møller energy and momentum prescription. This is because of the above mentioned important results obtained in the Møller complex and, also of the Lessner [13] opinion and Cooperstock's very important hypothesis [63].

3 Calculation of Gravitational Energy-Momentum

The aim of this section is to evaluate the energy and momentum distributions associated with the electric and magnetic black holes, using the Møller prescription in general relativity and teleparallel gravity, respectively.

3.1 Energy-Momentum in Einstein's Theory of General Relativity

In general relativity, the definition of the energy-momentum complex is given as [11, 12]

$$\Omega_\mu^\nu = \frac{1}{8\pi}\frac{\partial \chi_\mu^{\nu\alpha}}{\partial x^\alpha} \tag{17}$$

satisfying the local conservation laws,

$$\frac{\partial \Omega_\mu^\nu}{\partial x^\nu} = 0 \tag{18}$$

where, the antisymmetric super-potential $\chi_\mu^{\nu\alpha}$ is

$$\chi_\mu^{\nu\alpha} = \sqrt{-g}\left\{\frac{\partial g_{\mu\beta}}{\partial x^\gamma} - \frac{\partial g_{\mu\gamma}}{\partial x^\beta}\right\} g^{\nu\gamma} g^{\alpha\beta}. \tag{19}$$

The locally conserved energy-momentum complex Ω_μ^ν contains contributions from the matter, non-gravitational and gravitational fields. Ω_0^0 is the energy density and Ω_a^0 are the momentum density components. The momentum four-vector definition of Møller is given by

$$P_\mu = \iiint \Omega_\mu^0 dx dy dz. \tag{20}$$

Using Gauss's theorem, this definition transforms into

$$P_\mu = \frac{1}{8\pi}\iint \chi_\mu^{0a}\mu_a dS \tag{21}$$

where, μ_a (where $a = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3, and P_0 gives the energy.

The required non-zero components of the super-potential of Møller, for the electric and magnetic black holes, respectively, are

$$ {}_{\mathrm{EBH}}\chi_0^{01}(r,\theta) = \frac{2M\sin\theta}{(1+\frac{2M\sinh^2\alpha}{r})^2}\left[1+2\left(1-\frac{M}{r}\right)\sinh^2\alpha\right], \tag{22} $$

$$ {}_{\mathrm{MBH}}\chi_0^{01}(r,\theta) = 2r\sin\theta\frac{(2M^2-Q^2)}{2(Mr-Q^2)}. \tag{23} $$

By substituting these super-potentials given above into (17), one gets the following energy distributions:

$$ {}_{\mathrm{EBH}}\Omega_0^0 = -\frac{M^2\sin\theta[-3r+2(M-2r)\sinh^2\alpha]\sinh^2\alpha}{2\pi(r+2M\sinh^2\alpha)^3}, \tag{24} $$

$$ {}_{\mathrm{MBH}}\Omega_0^0 = \frac{(2M^2-Q^2)Q^2\sin\theta}{8\pi(Mr-Q^2)^2}, \tag{25} $$

while the momentum density distributions take the form

$$ {}_{\mathrm{EBH}}\Omega_1^0 = 0, \qquad {}_{\mathrm{MBH}}\Omega_1^0 = 0, \tag{26} $$

$$ {}_{\mathrm{EBH}}\Omega_2^0 = 0, \qquad {}_{\mathrm{MBH}}\Omega_2^0 = 0, \tag{27} $$

$$ {}_{\mathrm{EBH}}\Omega_3^0 = 0, \qquad {}_{\mathrm{MBH}}\Omega_3^0 = 0. \tag{28} $$

Therefore, if we substitute these results into (20), we get the energy distributions of the EBH and MBH that are contained in a *sphere* of radius r

$$ E^{\mathrm{EBH}} = \frac{M}{(1+\frac{2M\sinh^2\alpha}{r})^2}\left[1+2\left(1-\frac{M}{r}\right)\sinh^2\alpha\right], \tag{29} $$

$$ E^{\mathrm{MBH}} = r\frac{(2M^2-Q^2)}{2(Mr-Q^2)} \tag{30} $$

which are also the energy (mass) of the gravitational field that a neutral particle experiences at a finite distance r. Additionally, we find that the momentum components are

$$ \overrightarrow{P}^{\mathrm{EBH}} = 0, \tag{31} $$

$$ \overrightarrow{P}^{\mathrm{MBH}} = 0. \tag{32} $$

3.2 Møller's Energy-Momentum in Teleparallel Gravity

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian space [64, 65]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [66] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [67] showed that Møller theory is a special case of Poincaré gauge theory [68–70].

In teleparallel gravity, the super-potential of Møller is given by Mikhail et al. [46] as

$$\Sigma_{\mu}^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\beta}\left[\Phi^{\rho} g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu}\xi^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu}\xi^{\sigma\rho\chi}\right] \tag{33}$$

where, $\xi_{\alpha\beta\mu} = h_{i\alpha} h^{i}_{\beta;\mu}$ is the con-torsion tensor and h_i^{μ} is the tetrad field, and defined uniquely by $g^{\alpha\beta} = h_i^{\alpha} h_j^{\beta} \eta^{ij}$ (here η^{ij} is the Minkowski spacetime). κ is the Einstein constant and λ is free-dimensionless coupling parameter of teleparallel gravity. For the teleparallel equivalent of general relativity there is a specific choice of this constant.

Φ_{μ} is the basic vector field given by

$$\Phi_{\mu} = \xi^{\rho}{}_{\mu\rho}, \tag{34}$$

and $P_{\chi\rho\sigma}^{\tau\nu\beta}$ can be found by

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_{\chi}^{\tau} g_{\rho\sigma}^{\nu\beta} + \delta_{\rho}^{\tau} g_{\sigma\chi}^{\nu\beta} - \delta_{\sigma}^{\tau} g_{\chi\rho}^{\nu\beta} \tag{35}$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_{\rho}^{\nu}\delta_{\sigma}^{\beta} - \delta_{\sigma}^{\nu}\delta_{\rho}^{\beta}. \tag{36}$$

The energy-momentum density is defined by

$$\Xi_{\alpha}^{\beta} = \Sigma_{\alpha,\lambda}^{\beta\lambda}, \tag{37}$$

where comma denotes ordinary differentiation. The energy is expressed by the surface integral

$$E = \lim_{r\to\infty} \int_{r=\text{constant}} \Sigma_0^{0\zeta} \eta_{\zeta} dS, \tag{38}$$

where η_{ζ} (with $\zeta = 1, 2, 3$) is the unit three-vector normal to surface element dS.

The general form of the tetrad, h_i^{μ}, having spherical symmetry was given by Robertson [71]. In the Cartesian form, it can be written as

$$\begin{aligned} &h_0^0 = iW, \qquad h_a^0 = Zx^a, \qquad h_0^{\alpha} = iHx^{\alpha}, \\ &h_a^{\alpha} = K\delta_a^{\alpha} + Sx^a x^{\alpha} + \epsilon_{a\alpha\beta} G x^{\beta}, \end{aligned} \tag{39}$$

where W, K, Z, H, S, and G are functions of t and $r = \sqrt{x^a x^a}$, and the zeroth vector h_0^{μ} has the factor $i^2 = -1$ to preserve Lorentz signature and the tetrad of Minkowski spacetime is $h_a^{\mu} = \text{diag}(i, \delta_a^{\alpha})$ where $(a = 1, 2, 3)$.

Using the general coordinate transformation

$$h_{a\mu} = \frac{\partial \mathbf{X}^{\nu'}}{\partial \mathbf{X}^{\mu}} h_{a\nu} \tag{40}$$

where $\{\mathbf{X}^{\mu}\}$ and $\{\mathbf{X}^{\nu'}\}$ are, respectively, the isotropic and Schwarzschild coordinates (t, r, θ, ϕ). In the spherical, static and isotropic coordinate system $\mathbf{X}^1 = r\sin\theta\cos\phi$, $\mathbf{X}^2 =$

$r\sin\theta\sin\phi$, $\mathbf{X}^3 = r\cos\theta$, we obtain the tetrad components as

$$_{\text{EBH}}{h_a}^{\mu} = \begin{pmatrix} \frac{i(1+\frac{2M\sinh^2\alpha}{r})}{\sqrt{1-\frac{2M}{r}}} & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{2M}{r}}s\theta c\phi & \frac{c\theta c\phi}{r} & -\frac{s\phi}{rs\theta} \\ 0 & \sqrt{1-\frac{2M}{r}}s\theta s\phi & \frac{c\theta s\phi}{r} & \frac{c\phi}{rs\theta} \\ 0 & \sqrt{1-\frac{2M}{r}}c\theta & -\frac{s\theta}{r} & 0 \end{pmatrix}, \tag{41}$$

$$_{\text{MBH}}{h_a}^{\mu} = \begin{pmatrix} \frac{i\sqrt{1-\frac{Q^2}{Mr}}}{\sqrt{1-\frac{2M}{r}}} & 0 & 0 & 0 \\ 0 & \sqrt{(1-\frac{2M}{r})(1-\frac{Q^2}{Mr})}c\theta c\phi & \frac{c\theta c\phi}{r} & -\frac{s\phi}{rs\theta} \\ 0 & \sqrt{(1-\frac{2M}{r})(1-\frac{Q^2}{Mr})}c\theta s\phi & \frac{c\theta s\phi}{r} & \frac{c\phi}{rs\theta} \\ 0 & \sqrt{(1-\frac{2M}{r})(1-\frac{Q^2}{Mr})}c\theta & -\frac{s\theta}{r} & 0 \end{pmatrix}, \tag{42}$$

where, $i^2 = -1$. Here, we have introduced the following notation: $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\phi = \sin\phi$ and $c\phi = \cos\phi$.

To find the super-potential of Møller, first we should calculate the required non-vanishing basic vector field Φ_μ and con-torsion tensor $\xi_{\alpha\beta\mu}$. After making some calculations [72, 73], the required non-vanishing components of $\xi_{\alpha\beta\mu}$ and Φ_μ are found as

$$_{\text{EBH}}\xi^0_{01} = \left[\ln\sqrt{\left(1-\frac{2M}{r}\right)\left(1+\frac{2M\sinh^2\alpha}{r}\right)^{-2}}\right]_{,r}, \tag{43}$$

$$_{\text{EBH}}\xi^1_{11} = -\left[\ln\sqrt{\left(1-\frac{2M}{r}\right)}\right]_{,r}, \tag{44}$$

$$_{\text{EBH}}\xi^1_{22} = -r\sqrt{\left(1-\frac{2M}{r}\right)^{-1}}, \tag{45}$$

$$_{\text{EBH}}\xi^1_{33} = {}_{\text{EBH}}\xi^1_{22}\sin^2\theta, \tag{46}$$

$$_{\text{EBH}}\xi^2_{21} = {}_{\text{EBH}}\xi^3_{31} = r^{-1}, \tag{47}$$

$$_{\text{EBH}}\xi^3_{32} = {}_{\text{EBH}}\xi^3_{23} = \cot\theta, \tag{48}$$

$$_{\text{EBH}}\xi^2_{33} = -\sin\theta\cos\theta, \tag{49}$$

$$_{\text{EBH}}\xi^2_{12} = {}_{\text{EBH}}\xi^3_{13} = \left[r\sqrt{1-\frac{2M}{r}}\right]^{-1}, \tag{50}$$

$$_{\text{EBH}}\Phi_1 = -\left[\ln\sqrt{1-\frac{2M}{r}}\right]_{r} + 2\left[r\sqrt{1-\frac{2M}{r}}\right]^{-1}, \tag{51}$$

$$_{\text{EBH}}\Phi_2 = \cot\theta \tag{52}$$

and

$$ {}_{\mathrm{MBH}}\xi^{0}_{01} = \left[\ln\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)^{-1}}\right]_{,r}, \tag{53}$$

$$ {}_{\mathrm{MBH}}\xi^{1}_{11} = -\left[\ln\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)}\right]_{,r}, \tag{54}$$

$$ {}_{\mathrm{MBH}}\xi^{1}_{22} = -r\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)}, \tag{55}$$

$$ {}_{\mathrm{MBH}}\xi^{1}_{33} = {}_{\mathrm{MBH}}\xi^{1}_{22}\sin^2\theta, \tag{56}$$

$$ {}_{\mathrm{MBH}}\xi^{2}_{21} = {}_{\mathrm{MBH}}\xi^{3}_{31} = r^{-1}, \tag{57}$$

$$ {}_{\mathrm{MBH}}\xi^{3}_{32} = {}_{\mathrm{MBH}}\xi^{3}_{23} = \cot\theta, \tag{58}$$

$$ {}_{\mathrm{MBH}}\xi^{2}_{33} = -\sin\theta\cos\theta, \tag{59}$$

$$ {}_{\mathrm{MBH}}\xi^{2}_{12} = {}_{\mathrm{MBH}}\xi^{3}_{13} = \left[r\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)}\right]^{-1}, \tag{60}$$

$$ {}_{\mathrm{MBH}}\Phi_1 = -\left[\ln\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)}\right]_{,r} + 2\left[r\sqrt{\left(1-\frac{2M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)}\right]^{-1}, \tag{61}$$

$$ {}_{\mathrm{MBH}}\Phi_2 = \cot\theta. \tag{62}$$

Next, substituting these results into (33), we obtain the non-vanishing required Møller's super-potentials $\Sigma_\mu^{\nu\beta}$ as following

$$ {}_{\mathrm{EBH}}\Sigma_0^{01} = -\frac{4M^2\sin\theta\sinh^2\alpha}{\kappa(r+2M\sinh^2\alpha)^3}[2(M-2r)\sinh^2\alpha-3r], \tag{63}$$

$$ {}_{\mathrm{MBH}}\Sigma_0^{01} = \frac{2M^2-Q^2}{\kappa(Mr-Q^2)}r\sin\theta. \tag{64}$$

Using above results in the energy integral (38), we find the following energies for the EBH and MBH, respectively.

$$ {}_{\mathrm{EBH}}E(r) = \frac{Mr^2}{(r+2M\sinh^2\alpha)^2}\left[1+2\left(1-\frac{M}{r}\right)\sinh^2\alpha\right], \tag{65}$$

$$ {}_{\mathrm{MBH}}E(r) = \frac{2M^2-Q^2}{2(Mr-Q^{2)}}r \tag{66}$$

and the momentum components are

$$ {}_{\mathrm{EBH}}\overrightarrow{P} = 0, \tag{67}$$

$$ {}_{\mathrm{MBH}}\overrightarrow{P} = 0. \tag{68}$$

4 Final Comments

In this paper, we evaluated the energy and momentum distributions (due to matter and fields including gravitation) associated with the electric and magnetic black hole using Møller's energy-momentum complexes in general relativity and teleparallel gravity. The energy distribution of the electric black hole depends on the mass M and the magnetic black hole energy distribution depends on the mass M and charge Q. We obtain the same energy and momentum distributions in both of these different gravitation theories.

Both in Einstein's theory of general relativity and teleparallel gravity we find the following energy and momentum components for the electric and magnetic black holes.

$$E^{\text{EBH}} = \frac{Mr^2}{(r+2M\sinh^2\alpha)^2}\left[1+2\left(1-\frac{M}{r}\right)\sinh^2\alpha\right], \tag{69}$$

$$\overrightarrow{P}^{\text{EBH}} = 0, \tag{70}$$

$$E^{\text{MBH}} = \frac{2M^2-Q^2}{2(Mr-Q^2)}r, \tag{71}$$

$$\overrightarrow{P}^{\text{MBH}} = 0. \tag{72}$$

In some special cases, electric and magnetic black holes are reduced to the well-known spacetimes whose energies have been already calculated.

Case (i). Special Limits for the Electric Black Hole

1. The electric black hole is easily reduced to the Minkowski spacetime in the limiting of $M \to 0$ (or without mass). From (69), the total energy becomes

$$E^{\text{EBH}} = 0. \tag{73}$$

Such a result has been expected, for in the Minkowski spacetime, there is nothing that contributes to the energy distribution.

2. The other limit is the $r \to \infty$ (large distances). This limit yields the energy distribution as

$$\lim_{r\to\infty} E^{\text{EBH}} = M + 2M\sinh^2\alpha. \tag{74}$$

3. The last limit is $\alpha \to 0$. In this limit, the line element (5) describes the Schwarzschild spacetime and the energy distribution is found as

$$E^{\text{EBH}} = M. \tag{75}$$

It depends on the mass of the black hole.

Case (ii). Special Limits for the Magnetic Black Hole

1. Taking the limit $Q \to 0$, the magnetic black hole is reduced to the Schwarzschild spacetime and the energy distribution is obtained

$$E^{\text{MBH}} = M. \tag{76}$$

2. The other limit is the $r \to \infty$ (large distances). This limit yields the energy distribution as

$$\lim_{r\to\infty} E^{\mathrm{MBH}} = M - \frac{Q^2}{2M}. \tag{77}$$

3. Taking $Q \to 0$ and then $M \to 0$ limits, it is reduced to the Minkowski spacetime model and the energy distribution is found as

$$E^{\mathrm{MBH}} = 0. \tag{78}$$

Such a result has been expected, for in the Minkowski spacetime, there is nothing that contributes to the energy distribution.

Furthermore, one can easily see that the results are independent of the teleparallel dimensionless coupling parameter λ. Hence we can say that these results are valid not only in teleparallel equivalent of general relativity but also in any teleparallel model. Our results also supports the viewpoint of Lessner that the Møller energy-momentum formulation is powerful concept to calculate energy and momentum distributions associated with the universe, and sustains the importance of the energy-momentum definitions in the evaluation of the energy-momentum distribution of a given spacetime.

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